A COMPARATIVE STUDY OF SHIP HULL FAIRING USING SPLINE AND BEZIER FUNCTIONS

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Abstract Three interpolating functions viz., cubic spline, B-spline and Bezier were used to investigate the effectiveness of the same for fairing lines plan of some inland/coastal vessels. The objective is to develop a mathematical tool to substitute the fairing system practiced in the shipyard loft floor. The fairing was carried out for individual sections. The cubic spline method was used because of the fact that this method is the mathematical version of the baten based method traditionally used in ship loft floor. The B-spline and Bezier functions were used because of the flexibility of these methods in fitting complex curves. The results of the faring procedure show that cubic splines are very useful in convex hull shapes and is reluctant to adopt a concave shape. This is apparently strongly related to the coordinate reference axes used for the calculation. Cubic splines are not at all appropriate for the double curvature shapes. However, such shapes are found beneficial in ship section at the aft end because of other functional reasons. Bezier and B-splines are found to be better suited for double curvature sections and concave sections but are found to be less efficient that cubic spline in convex sections.

1. INTRODUCTION

In the field of surface modeling of hull form, geometric complexity of hull form gives many difficulties in adopting surface modeling technique, which can describe the irregular topological characteristics precisely. Many methods have been developed to generate curves and surfaces from a set of data points or one group of parameter curves [1-3]. The earlier ones, such as cubic splines, Coons patches, Gordon’s patches and splines in tension, interpolated all the defining points. In general, their main drawbacks are their global behavior that implies that any local changes affect the complete shape. Also a problem is dealing with some quantities, such as cross-derivatives and others, whose influence in the shape is not obvious for the designer. The use of Bezier curves and surfaces introduced the concept of control polygons and meshes that provide a more intuitive geometric control of the shape. To smooth existing surface, a lot of methods have been applied; for example, mesh fairing method [4], reflection line method [5], FANGA curves method [6], minimizing a sum of the squares of principal curvatures [7] etc.

Generation of faired hull surface is also very important in ship production procedure. Ship drawings are prepared in scales of 40:1 to 100:1. During the construction phase, full-scale drawings are prepared at the so-called “loft-floor”. In this process, the minute discontinuity in the shape of the vessel that is not apparent in the scaled drawings becomes apparent. This requires a modification of the drawings that is both tedious and time consuming. Computer based techniques are being evolved to measure the continuity of the shape and thus to ensure that the scaled drawings are truly continuous avoiding the necessity of the corrections from the lofting process. Such an early fairing and modeling of ship hull in preliminary design stage has more effective advantages in many sides, for example, performance analysis, production design and process management etc.

As such, no universal tools have yet evolved for carrying out these procedures. As described above, many techniques have been tried with varying degrees of success and limitations. Various criteria for fairness have been considered but none without limitations and universal applicability. Various versions of splines and other types of curve fitting techniques have also been used. The present study aims at investigating the effectiveness of the application of cubic spline, Bspline and Bezier functions [8] in curve fitting and fairness measurement as applied to ship design.

2. METHODOLOGY

The process of ship lofting is carried out in the loft floor using battens; the physical representation of the cubic splines. The transverse sections are first faired using the battens. The battens are made to pass over the data points representing the section. The data points are available from the offset table, the breadth at the waterplanes and the height above base line. The fairing is carried out in the procedure described below.

To make the baten to pass over the data points, weights are applied at such points. Otherwise, the baten will be a straight one. The two ends of an individual section are held fixed. The weight applied over one of the other data points is removed with other weights in place. If the baten shifts in any direction by a significant distance due to removal of the weight, it indicated that the curve is not smooth and it has severe discontinuities in the first or second derivatives. In order to smoothen the shape, it must be allowed to shift in the direction the baten has shifted at the data point where the weight was removed. This process is continued sequentially on all data points except at the two ends.

2.1 Cubic Spline

In fact, fitting a cubic spline and passing a baten through a set of data points are exactly identical exercises. While a baten is made to pass through a set of data points, the baten will pass without breaking if the second derivatives of the curves at the data points are continuous. Splines will pass over any set of data points with unique values of x for y. However, spline not necessarily all splines will be smooth since the fitting may be a forced one. Mathematically, this is a situation where the moment at any data point is a very high one. Thus the fairing procedure is executed until the moment at the data points is minimum one.

However, there is a difference between spline fitting done physically with the help of batten and mathematically using the procedure explained above. The coordinate system in the physical process adapts automatically to the characteristics of the input data set. On the other hand, in the mathematical system, the computations have to be performed with reference to an assumed Cartesian coordinate system. This assumption plays a vital role in shaping the spline. For example, if we take the axes either horizontal or vertical, it is very difficult, and in some cases impossible, to fit the curves.

Considering these situations, the coordinate system has been assumed with the axes at 45^0 with the vertical or horizontal.

2.2 B-spline and Bezier

Since the B-spline and Bezier functions are bit different from cubic spline in nature, it is bit difficult to correlate the mathematical model with the physical fairing process. However, it must be remembered that if the input points are smooth, the Bezier or B-spline will pass over the input data points. The process adopted consisted of interpolating the Y value corresponding to the various X values. Unlike the cubic spline function, the data point corresponding to the point under investigation was not deleted from the input data set. It may be remembered that in the B-spline and Bezier functions, the second and third data points works as a weighing function and thus inclusion of the data set under investigation in fact insists the B-spline or Bezier to pass over the same. Thus both a fair curve is obtained along with the guideline on where the curve should pas through. The same coordinate system as adopted in the case of cubic spline was used.

3. APPLICATION IN SHIP FAIRING PROCEDURE

As customary in the loft floors in shipyards, the fairing in this case is also performed in the athwart sections. For the purpose of fitting and fairing with cubic splines, the various sections have been divided into four categories as below:

Category I: Sections where some segment both at the deck end and the keel ends are straight.
Category II: Sections where some segment at the deck end only is straight and but the keel end is smooth curve, not straight.
Category III: Sections where some segment at the keel end only is straight and but the deck end is

smooth curve, not straight.

Category IV: Sections where both the deck end and the keel end are not straight.

In the context of the present study, it may be emphasized that the objective is to generate a shape of the hull that will be fair and at the same time will possess geometric characteristics closest possible to the input hull shape. For this reason, it has been decided that the segments of the hull, which has been designed, as straight will be maintained so, and no attempt for fairing this region will be carried out. This also defendable due to the fact that the straight are already fairest in shape. Also considering that we are using cubic spline, any attempt to implement fairing process in the straight region could result in either divergence of the procedure or may produce a 'fair' hull, which is not functionally desired.

It may be recalled that in case of the cubic spline as well as the B-spline and Bezier function, in addition to the input data some other assumptions is also required. In the case of cubic spline, this is generally assumed in terms of the first or second derivative of the curve at the two ends. In the case of B-spline and Bezier functions, two extra data points are required at the ends. In one way, the assumptions are implicitly of similar nature because it is the extra data points at the two ends, which dictate the slope at the ends of the curve. These assumptions as used in the present work are explained below.

3.1 Cubic spline

The following boundary conditions are applied in all the categories,

1) Should the section have straight end(s) (either or any of the deck or keel ends) the slope of the straight segment will be same as the slope at the respective ends. This is understandable since the straight segments must have continuity with the rest.

2) The ends of the section where there is no straight end, the second derivative will be zero, i.e., the first derivative will be constant and the end will be straight at the specific end. This is because the sections are not curved at any of the end points.

3.2 B-spline and bezier function

1) The ends of the section where there is no straight end, the extra data points are assumed same as the data at the end of the section.

2) The ends of the section where there is a straight segment, the data of the preceding sections are used.

4. RESULTS AND DISCUSSIONS

The objective of the present work is to examine the effectiveness and appropriateness of cubic splines, B-splines and Bezier functions as a tool for fairing the lines of a ship. The exercise is also expected to point out the various bottlenecks, difficulties and special situations arising out in attempts to use these mathematical functions for fairing of ship lines. As explained earlier, cubic splines is apparently the mathematical version of the fairing process carried out in shipyard loft floor. In order to do the samethe following vessels were selected randomly and the lines plan of the same were tested by the programs developed.

1) 52.00 meter Oil Tanker
2) 16.80 meter Tugboat

The principal particulars are of the vessel are given in Table1.

Table –1 Principal Particulars of the subject vessels

Vessel	Length (m)	Breadth (m)	Depth (m)	CB	CM	CW
Oil Tanker	52.00	9.30	4.60	0.743	0.990	0.877
Tugboat	16.80	3.20	1.90	0.676	0.907	0.900

Both of the vessels are existing ones. It is expected that the lines plans of these vessels are faired one using the conventional techniques. While it is not wise to assume that the lines are perfectly fair, it can not be either firmly advocated that these are extremely faulty is any mathematical process indicate so. Thus in this situation the philosophy adopted is that we assume the lines to be by and large fair but mathematical fairing may point out small regions of breaks of continuity and at the same time if the mathematical model produces a faired curve much different from the input, we should look at any probable reason for this disagreement and try to identify where the mathematical model itself should be improved.

The process of fairing of the lines plan of each of the vessels under study was carried out using cubic spline, Bezier function and B-splines and compared with the input. The results are discussed in the following.

4.1 Cubic Spline

4.1.1 Coastal oil tanker

Figure 1 shows the shapes of the unfaired and faired sections using cubic spline of the coastal oil tanker. It may be observed from the drawings that most of the convex sections (refer to the coordinate system used for the mathematical model) becomes fair without much change. However, the sections with double curvature seem to be reluctant to get faired without much alteration, especially with a tendency to eliminate the double curvature. This is obvious since a baten will not very easily fit into such a shape. This is also important to observe that such sections appear in the aft of the vessel and is generally used to simultaneously attain three functional objectives in a ship

(i) ensure best possible wake field in the propeller plane,
(ii) ample deck space at the aft and
(iii) attaining a desired longitudinal center of buoyancy of the vessel

However, this double curvature sections do not seriously impair the operation of the vessel since the prime consideration in such sections is the flow of water in the longitudinal direction, i.e., the characteristics of flow into the propeller section that is properly achieved by the section employed. However, the double curvature takes its toll in real operating condition with the velocity component of water in vertical direction. The designer has to make a compromise between the alternatives so as to achieve the ultimate benefit satisfying the various functional requirements. Thus although the sections are not at all fair from as far as cubic splines are concerned, such sections are employed in real cases and any recommendation to amend the shape must be based on the factors influencing the functioning of the vessel in real operating condition. Alternatively, the cubic spline fairing may be improved by employing techniques that itself make a compromise between a desired shape and smoothness of the shape. The technique employed in the present study is based exclusively on the properties of cubic spline and, as discussed earlier that the function is a global one and an alteration in a single data causes the entire curve over all segments to change its shape.

An examination of the fore sections of the vessel shows that the in the middle section of the hull, the fairing process has caused the shapes to shift inward at the bilge sections. It may again be noted that the subject vessel is an oil tanker and the economy of the oil tanker operations is inclined positively in favor of such shapes and thus oil tanker always have a higher block coefficient than other cargo vessels. Thus there exists a conflict between a properly faired hull shape and economic hull shape. In other words, the economic benefits of a fuller shape are more than the economy achieved by making the bilge fuller. Again, a change in the approach of the fairing procedure can, perhaps, be made which will consult the physical conditions as well rather than the theory of the splines.

In the fore sections, very little appears to have been done to make them fair. However, the double curvature effect generated by the bow flares appears to have made cause some shift in the section. It may be noted that such fairs are required for achieving proper sea-keeping characteristics. Again there is a contradiction between a shape, which is required for functional purpose and one certified as fair by the cubic spline function. The cubic spline technique may be improved as suggested earlier.

4.1.2 Tug Boat

The body plan of the tugboat as shown in Figure 2 appears to be better suited to the theory of faring by cubic spline. The response of the double curvature sections are similar and, apparently, for same reason. However, the functional requirement in this case is slightly different. Tugs are fitted with higher power engines compared to the size of the vessel. Thus the propeller diameter is large. This necessitated in the aft section to be sufficiently fine. But this is somewhat reduced by the fact that most tugboats have at least two engines and thus the designer can afford to make the stern relatively fuller maintaining proper inflow of water into the propeller plane. The bilge zone in the mid section of the vessel has been shifted considerably inward due to the fairing process. The input shape in the fore section is smooth by cubic spline theory and a slight change has been dictated in section No. 13.

4.2 Fairing Using Bezier Function

It may be recalled that Bezier function is a local one. The computations are made using just four of the data points and a change in one data does not affect the curve two segment apart. Thus it is expected to be more adaptive to changes in shapes and an undulation is not expected to cause a major shift in other places in the curve.

4.2.1 Coastal Oil Tanker

Fig 3 shows the various unfaired and Bezier faired sections of the oil tanker. The vessel is the same one as presented and discussed for fairing employing cubic splines. The results show that, as in the case of cubic splines, the sections with double curvature (sections at the aft) tends to get flatter and thus avoid the double curvature. However, there are some deviations. For example, the section No. 3 has flattened slightly although the extent of double curvature is quite strong. On the other hand, the Section No. 6 has become rather straight after being faired. In section No. 6 and 7, the convex portion has not altered due to fairing but the concave section has become almost straight. However, unlike in the case of cubic splines, the Bezier seems to have responded amicable to the slight double curvatures in the forward sections (Section No. 18 and 19).

The faired curved and unfaired curves in the other sections are also closed compared to the results produced by the cubic splines. The inward shift in the section of the bilge is also smaller in this case. It is expected that some further improvements and refinement in the method could, perhaps, lead to a better result.

4.2.2 Tugboat

The unfaired and Bezier faired shapes of the various sections of the vessel are shown in Fig 4. The double curvature sections are more amenable to the fairing process compared to the response in the cubic spline. Double curvature natures has remained in section No, 3 and 4. The slightly concave segment near the keel end has been flattened. The shift in the bilge segment in Section No. 10 is moderate and very small in Section 12. Section No. 11 has shown slightly different response. In section lower than bilge, the convexity has increased whereas in segments above the bilge, the convexity has decreased. However, contrary to expectations, Section 13 and Section 15 has altered slightly as a result of the fairing process although both the sections did have apparently a smooth shape before the fairing. Even in the cubic spline which fair the curve on a global basis, the faired curve was very close to the unfaired one.

4.3 Fairing Using B-Spline

It may be recalled that B-spline function is a local one. The computations are made using just four of the data points and a change in one data does not affect the curve two segment apart. Thus it is expected to be more adaptive to changes in shapes and an undulation is not expected to cause a major shift in other places in the curve as in the case of Bezier functions.

4.3.1 Coastal Oil Tanker

Fig 5 shows the various unfaired and B-spline faired sections of the oil tanker. The vessel is the same one as presented and discussed for fairing employing cubic splines and Bezier. The results are similar to that of the Bezier. The results show that, as in the case of cubic splines and Bezier, the sections with double curvature (sections at the aft) tends to get flatter and thus avoid the double curvature. The deviation as found in the case of Bezier function, the section No. 3 has flattened slightly although the extent of double curvature is quite strong. On the other hand, the Section No. 6 has become rather straight after being faired. In section No. 6 and 7, the convex portion has not altered due to fairing but the concave section has become almost straight. However, unlike in the case of cubic splines, the both Bezier and B-spline seems to have responded amicable to the slight double curvatures in the forward sections (Section No. 18 and 19).

The faired curves and unfaired curves in the other sections are also closed compared to the results produced by the cubic splines. The inward shift in the section of the bilge is also smaller in this case compared to the cubic spline. It is expected that some further improvements and refinement in this method also could, perhaps, lead to a better result.

4.3.2 Tugboat

Fig 6 shows the various unfaired and B-spline faired sections of the tugboat. The vessel is the same one as presented and discussed for fairing employing cubic splines and Bezier. The results show that, unlike the case of cubic splines or Bezier, the sections with double curvature (sections at the aft) does not tend to get flatter and thus avoid the double curvature. The faired sections do have strong double curvatures. Most of the aft sections have, however, experienced some changes in the bottom side and the faired curve is fuller compared to the input (unfaired). The shape has moved outward in the bilge section in the middle body of the vessel. The section No. 11 has experienced considerable change in shape but perhaps not the area under the curve has changes significantly. Section 14 which is slightly convex in shape has shifted slightly in the outward directions. The concave section 16 has not changed as a result of the fairing process.

5. CONCLUSIONS

The study shows that piecewise polynomials such as cubic spline, B-spline and Bezier functions can be useful in defining and fairing lines plan of ships. However, there are both limitations and advantages with each of these methods. The cubic spline has been found to be very effective in defining and faring convex sections but is reluctant to adopt a faired concave shape. The function is very poor in handling curves with double curvature. The cubic spline, when asked to fair a section with double curvature, tends to straighten the same. This is because of the global nature of the method. The B-spline and Beziers functions are better in handling sections with double curvature. These methods are better to cubic spline in fairing concave sections. These merits of the B-splines and Bezier functions are apparently due to the local nature of the functions; the impact of a single input data set does not affect the entire curve. However, the B-spline and Bezier functions are found less effective in fairing convex sections.

It is felt important that some improvements and amendments to the cubic spline, B-spline and Bezier functions as adopted in this study may be made so as to ensure that the faired curve do have minimum alteration from the input data. This is particularly important because a desired fair hull should also have characteristics that satisfy the other requirements of a ship hull especially in terms of displacement, position of longitudinal center of gravity, water inflow into the propeller plane, adequate deck area etc.

Since construction of piecewise polynomials such as cubic splines, B-splines and Bezier require some extra input in addition to the data points, the assumptions on these extra input greatly influence the nature of the constructed splines and the results of the fairing procedure. In some cases, it is seen that the slope and curvature at the ends of the section has been greatly changed due to the fairing procedure. The present works can be improved to ensure proper slope and curvature at the ends. Such an improvement is expected to result in a fair hull that will be close to the desired shape of the vessel from other considerations.

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